Chapter 21: Accumulation and Reproduction on an Expanded Scale

Marx’s intentions in laying out the schemes of expanded reproduction of social capital which are the central topic of this chapter, for all the subsequent fascination they have attracted, are not immediately transparent. Is it the case that Marx wished to paint a ‘picture of an economy in perpetual balanced growth’,¹ of a capitalist reproduction in perpetual equilibrium? Or does he want to portray the conditions for equilibrium in social reproduction so as to demonstrate their practical unattainability?² Is Marx painting a picture of an inevitably crisis-free capitalism, or an inherently crisis-ridden one?

In good part, Marx’s inspiration for the reproduction schemes came from the impact made on him by Quesnay’s Tableau Économique,³ ‘incontestably the most brilliant [conception] for which political economy had up to then been responsible.’⁴ In Marx’s words, what Quesnay wanted to portray was

> the whole production process of capital as a *process of reproduction*, with circulation merely as the form of this reproductive process; and the circulation of money only as a phase in the circulation of capital; at the same time to include in this reproductive process the origin of revenue, the exchange between capital and revenue, the relation between reproductive consumption and final consumption; and to include in the circulation of capital the circulation between consumers and producers (in fact between capital and revenue); and finally to present the circulation between the two great divisions of productive labour – raw material production and manufacture – as phases of this reproductive process.⁵

It does not require a too far stretch of the imagination to conceive of Marx wanting to achieve something similar.⁶ And, in so doing, prominent in his mind was his critique of Adam Smith’s ‘dogma’⁷ that the value of commodities in its totality could be ‘resolved’ into ‘revenue’, that is profit and wages.⁸ If this were true, argued Marx, the total social product would be consumed (by capitalists and workers) and there would be no accumulation: ‘society would have to start each year *de novo*, without capital.’⁹ But if it were not true, if a part of the product was not consumed, but *accumulated*, then the problem to be solved was the apparent imbalance between revenue and product, between what was consumed and what was produced. How, if what was spent as ‘revenue’ was necessarily less than the value of what had been produced, would that part of the product to be accumulated be sold?

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⁵ *Theories of Surplus-Value* vol. 1, pp. 343-4.

⁶ See ‘Marx To Engels In Manchester’ (6 July 1863), <http://www.marxists.org/archive/marx/works/1863/letters/63_07_06.htm>.

⁷ Marx’s word: ‘Adam Smith put forward this fanciful dogma [...] according to which the entire value of the social product resolves itself into revenue, i.e. into wages plus surplus-value, or as he expresses it, into wages plus profit (interest) plus rent. He also put it forward in the still more popular form that the consumers must ultimately pay the producers for the entire value of the products. Right to the present, this remains one of the most well-loved platitudes, or rather eternal truths, of the so-called science of political economy.’ C2, p. 510.

⁸ ‘[T]he most important immediate purpose of Marx’s reproduction tables was to refute the widely-held, but erroneous, view of Adam Smith that the price of the total social product is entirely resolved into revenue, that is, into wages plus profit and rent.’ Fred Moseley, ‘Marx’s Reproduction Schemes and Smith’s Dogma’, in Christopher J Arthur and Geert Reuten (eds.), *The Circulation of Capital: Essays on Volume Two of Marx’s Capital* (London and New York, 1988), p. 160.

⁹ ‘Marx To Engels In Manchester’ (6 July 1863).
How is it [...] possible for the worker with his wages, the capitalist with his profit, the landowner with his rent, to buy commodities that contain not only one of these components [constant capital, variable capital and surplus-value] but all three, and how is it possible for the value sum of wages, profit and rent, i.e. the three sources of income taken together, which are to buy the commodities which are to enter into the total consumption of the recipients of these incomes, to contain a further additional value component on top of these three, i.e. constant capital? How can a value of four be bought with a value of three?10

Marx’s expanded reproduction schemes show that capitalists are not only able to recover the constant capital component of the social product (so as to continue production on the same scale) but also to accumulate more of the social product as new capital (both constant and variable). To do this, Marx divides total social production into two departments: that which produces means of production (means of productive consumption), and that which produces means of subsistence (means of unproductive consumption). It is important to grasp that this distinction is itself a product of one of the fundamental specificities of capitalist production: the separation of the worker from the means of production, a separation of which the separation between means of production and means of subsistence is a consequence and reflection:11

[...] the conditions for the realization of labour-power, i.e. means of subsistence and means of production, are separated, as the property of another, from the possessor of labour-power. [...] The capital relation arises only in the production process because it exists implicitly in the act of circulation, in the basically different economic conditions in which buyer and seller confront one another, in their class relation. [...] If the sale of one’s own labour-power (in the form of the sale of one’s own labour, or the wage form) is not an isolated phenomenon, but the socially decisive precondition for the production of commodities [...] this [...] implies the occurrence of historic processes through which the original connection between means of production and labour-power was dissolved; processes as a result of which the mass of the people, the workers, come face to face with the non-workers, the former as non-owners, the latter as the owners, of these means of production. [...] Thus the situation that underlies the act \( \text{M} - \text{C}_L^< \text{mp} \) is one of distribution; not distribution in the customary sense of distribution of the means of consumption, but rather the distribution of the elements of production themselves, with the objective factors concentrated on one side, and labour-power isolated from them on the other. [...] We have already seen how capitalist production, once it is established, not only reproduces this separation in the course of its development, but also expands on an ever greater scale until it has become the generally prevailing social condition.12

Thus ‘Marx’s main purpose in developing the schemes was not to model balanced growth – nor to model unbalanced growth’,13 but to depict how it is possible for reproduction to occur: how, if the value of the social product is greater than the value of the factors of production paid for to produce it, it is possible to realise this product (i.e. convert it into money), not only once, but repeatedly, and on an increasingly expanding scale. Inter alia, it is also possible to read into Marx’s account a description of the difficulties inherent in balanced reproduction – and by ‘balanced reproduction’ what is meant here is that, within the assumptions of the model, all the commodity product is realised such that optimum expanded reproduction occur in the next production period – for the conditions necessary to realise this are so precise that it is impossible to see how they could be reproduced within the circumstances of the ‘competition of many capitals’; demonstration of this point, however, does not appear to be Marx’s principal purpose. The reproduction schemes should be read with this in mind.

My notes here are in four parts. The first two parts, following Marx’s text, deal with specific questions related to the mechanisms of accumulation in each of the two departments of production, and then recapitulate, with a commentary, Marx’s numerical schemes; the third part is a reflection on the way that the patterns of accumulation in each department impact on those of the other; the final part is a reflection on issues not directly addressed in the schemes.

12 C2, pp. 114-7.
II  Accumulation in each department

1  The balance of sales and purchases

We begin by looking at accumulation in department I.

The two ways of expanding production are the expansion of functioning capital, and the setting up of new businesses. To do this, money capital, gained through the realisation of surplus-value, is hoarded, and then converted into means of production. Evidently, while some capitalists are hoarding, others are converting. The two groups of capitalists (of department I) thus conceived confront each other as sellers and buyers respectively.

\[ A \text{ sells } 400_c + 100_v + 100_s \text{ to } B, \text{ M100}^{15} (= \delta) \text{ is hoarded, i.e. withdrawn from circulation, and thus represents an } \]

obstacle to circulation.\(^{16}\) When capitalist \( A, A', A'' \), etc. hoard, they sell without buying; \( B, B', B'' \), etc. cast money into circulation and withdraw commodities, constant capital, both circulating and fixed.

We need to remember here that the circulation of the social product – including the reproduction of capital – does not consist in simple Say's law balances of sales and purchases, simple exchanges (barter) of one commodity for another. As we saw in the last chapter,\(^{17}\) the renewal of the fixed capital component of II,\(^{17}\) supposes unilateral purchases for that part of the fixed capital to be renewed in kind and unilateral sales for the other to realise the depreciation fund precipitated out in money form. Our assumption, so that reproduction not be disrupted, is that ‘one-sided purchase at one point must be covered by and one-sided sale at another.’\(^{18}\) We assume that, in \( I \), the hoard-forming sales of \( A, A', A'' \), etc. balance the one-sided purchases made by \( B, B', B'' \), etc.

To the extent that unilateral buyers later make unilateral purchases, and \textit{vice versa}, Marx comments that ‘the real balance [...] as far as the actual commodity exchange is concerned, i.e. the reconversion of the various parts of the annual product, requires that equal values of commodities are reciprocally exchanged.’\(^{19}\)

Expanding on this, Marx makes a point of general methodological significance with respect to his reproduction models and their interpretation. He notes that unilateral exchanges are inevitable in capitalist production, and he notes that, in capitalist production, in addition to means of circulation, money also functions as money capital in the circulation sphere. Then he says that, as a result, there exist ‘conditions for the normal course of reproduction’, conditions peculiar to the capitalist mode of production, ‘which turn into an equal number of conditions for an abnormal course, since on the basis of the spontaneous\(^{20}\) pattern of this production, this balance is itself an accident.’\(^{21}\)

2  The mediation of exchanges between departments

Marx now introduces another illustration of the difficulty of achieving the necessary conditions for reproduction, and how balance in exchange expresses itself. The exchange between capitalists (I) and capitalists (II) for the value

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\(^{14}\) The subheads in this chapter are mine; I have decided, for reasons of clarity of exposition, to eschew Marx’s (or Engels’) subheads altogether.

\(^{15}\) As in the previous chapter, prefix M indicates money of an unspecified currency.

\(^{16}\) Marx adds here (C2, p. 569) that ‘the quantity of money present in society is always greater than that part in active circulation.’ He also notes that what the credit system does is to concentrate these hoards, as disposable – ‘loanable’ – capital.

\(^{17}\) C2, pp. 524-45.

\(^{18}\) C2, p. 570.

\(^{19}\) C2, p. 570.

\(^{20}\) ‘\textit{Naturwüchsigen}’ – naturally occurring, occurring of its own accord and according to its own inherent natural dynamics.

\(^{21}\) C2, p. 571. This comment, that it is necessary to fulfil conditions which could only be fulfilled by chance, i.e. that each ‘condition’ is not only a precondition for order but also an obstacle to it, lends weight to the interpretation of the reproduction schemes not as a description of how capitalist reproduction \textit{does} take place, but how capitalist reproduction \textit{would} have to take place if it \textit{were} to take place in a balanced way and untroubled by crisis.
components I_\text{I} and II_\text{I} is indirect: it is mediated by the necessary first step of sale of commodities on the part of capitalists (II) to workers (I). Capitalist (II), in order to be able to buy commodities (means of production) from capitalist (I), needs first to sell commodities (means of consumption) to workers (I), i.e. needs first to convert the commodity value II_\text{c} into money form equivalent to I_\text{v}, like this (commodity movements in blue, displacement of money in red):

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{The mediation of the exchange I_\text{v} \leftrightarrow II_\text{c}}
\end{figure}

Hence, behind I_\text{v} \leftrightarrow II_\text{c} lies a set of conditions – a continuous supply of labour-power in I, the transformation of a part of I’s commodity capital into the money form of \( r \), the replacement of a part of II’s commodity capital in kind – ‘necessary preconditions [which] mutually require one another, but [which] […] are mediated by a complex process which involves […] processes of circulation that proceed independently […]'. The very complexity of the process provides many occasions for it to take an abnormal course.}\textsuperscript{22}

3 The source of the additional constant capital in department I

The surplus product is the repository of surplus-value; initially in the hands of \( A, A', A'' \), etc. this surplus product exists in the natural (use-value) form of means of production: constant capital for \( B, B', B'' \), etc., but as yet only potentially. ‘If we simply consider the level of reproduction on the part of department I in value terms, then we still find ourselves within the limits of simple reproduction. […] The distinction [between simple and expanded reproduction] here lies only in the form of the surplus labour applied […]',\textsuperscript{23} i.e. on whether it has been spent on means of production for department I (expanded reproduction) or on means of production for department II, i.e., on means of production of means of production or means of production of means of consumption.

In the case of simple reproduction, […] the whole of the surplus-value in department I was spent […] on commodities from department II; it consisted only of those means of production needed to replace the constant capital II_\text{c} in its natural form. […] [I]n order to make the transition from simple reproduction to expanded reproduction, production in department I must be in a position to produce fewer elements of constant capital for department II, but all the more for department I.\textsuperscript{24}

Considering expanded reproduction exclusively from the point of view of value, its ‘material substratum’\textsuperscript{25} – the surplus labour spent in department I on the production of means of consumption – already exists within simple reproduction.

The sale of this surplus product under conditions of expanded reproduction leads to the formation of ‘virtual’\textsuperscript{26} –

\textsuperscript{22} C2, p. 571.
\textsuperscript{23} C2, p. 572.
\textsuperscript{24} C2, p. 572.
\textsuperscript{25} C2, p. 573.
\textsuperscript{26} C2, p. 573.
potential – money capital, but the accumulation of this potential money capital does not represent an increase in the amount of money in existence: all that has happened is that money already in circulation assumes a different function, now that of a hoard. Hence ‘[t]he formation of additional money capital and the quantity of precious metals existing in a country […] do not stand in any causal connection with one another.’

Another conclusion that flows from these observations is this. The greater the quantity of functioning productive capital, and the higher the level of productivity, and hence the greater the amount of surplus-value, the greater (1) the additional potential capital in the form of surplus product in the hands of A, A', A'', etc., and (2) the quantity of this product transformed into money.

The surplus product produced (and appropriated) by the capitalist of department I ‘is the real basis for capital accumulation.’ Nevertheless, this product, as a hoard, formed piecemeal, is ‘absolutely unproductive in its monetary metamorphosis’: as a hoard it lies not inside the system of production, but alongside it. It is on the basis of this hoarded surplus that the credit system arises.

With the increase in the quantity of productive capital functioning the absolute quantity of surplus product in money form also grows. This increase in size facilitates is ‘segmentation’ – i.e. its separation from its parent capital, allowing it to be invested in independent businesses.

As we have seen, the surplus product, potential productive capital, really functions as such in the hands of B, B', etc. (department I), and not in those of A, A', etc.: capitalists B, B', etc. need to buy the surplus product from A, A', etc., i.e. it needs to enter into circulation. We need to note here that this additional constant capital, when it passes from A, A', etc. to B, B', etc., will, in its greater part, enter into production in the next, or even a subsequent, production period.

To the extent that the product of B, B', etc. enter in kind into their own production process, B, B', etc. cannot realise the surplus of A, A', etc.

When looking at simple reproduction, we saw that the capitalists of both departments were able to exchange their surplus product because of money that already existed in their hands. In the case of expanded reproduction, the money that would have been spent as revenue now returns to the capitalists to the degree that it has been advanced for the exchange of their respective product; i.e. the money is the same, but the function is different. ‘The As and Bs (department I) supply one another with the money for transforming their surplus products into additional virtual money capital, and alternately cast the newly formed money capital into the circulation sphere as a means of purchase.’ The only thing that needs to be assumed here is that the quantity of money in existence is sufficient (1) to maintain circulation; and (2) to maintain the reserve hoards. As capitalist production develops, the quantity of money necessary also grows, because (1) more and more products are produced as commodities; (2) as the mass and value of commodity capital grow so does its rate of growth; (3) the variable capital to be converted into money capital is increasingly extensive (proportionate growth of the proletariat); (4) of the proportionate growth of new money capital along with production itself.

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27 Except, Marx notes, when the buyer of the product is a producer of precious metals, i.e. of money itself.
28 C2, p. 573.
29 C2, p. 574, although it will only really function as such in the hands of B, B', B'', etc. (department I).
30 C2, p. 574.
31 C2, p. 575.
32 Marx’s next comment (C2, p. 576) is methodologically significant for the theoretical accommodation of fiat money. ‘If this [i.e. the growth in the necessary quantity of money with the development of capitalist production] is true absolutely for the early phase of capitalist production, where the credit system is accompanied by a predominantly metallic circulation, it is just as true, too, for the most developed phase of the credit system, which still has metallic circulation as its basis. On the one hand, the extra production of precious metals, according to whether this makes them abundant or scarce, can now exert a disturbing influence on the price of commodities, not only in the long term, but also within very short periods; on the other hand, the whole credit mechanism must constantly be engaged in restricting the actual circulation of metal by all kinds of operations, methods, technical devices, to what is relatively an ever decreasing minimum – though this also increases in the
4 The source of additional labour-power in department I

Marx suffices by noting that in volume 1 it had been observed\(^\text{33}\) that ‘labour-power is always on hand, and […] if necessary, more labour can be extracted without an increase in the number of workers employed, or the mass of labour-power.’\(^\text{34}\)

II Expanded Reproduction of Social Production

1 The importance of the quantitative relation between \(I(v + s)\) and \(IIc\)

Marx sets out two cases:

\[
\begin{align*}
&\text{(a)} \quad I \quad 4,000c + 1,000v + 1,000s = 6,000 \\
&\text{II} \quad 1,500c + 376v + 376s = 2,252 \\
&\text{total: 8,252} \quad \text{\textsuperscript{35}}
\end{align*}
\]

\[
\begin{align*}
&\text{(b)} \quad I \quad 4,000c + 875v + 875s = 5,750 \\
&\text{II} \quad 1,750c + 376v + 376s = 2,502 \\
&\text{total: 8,252}
\end{align*}
\]

We need to remember here that a condition for simple reproduction is \(I(v + s) = IIc\).

In case (a), \(I(v + s) = (1,000v + 1,000s)I = 2,000\), and \(IIc = 1,500\), so there is a surplus of 500\(I\) for accumulation in department I. In case (b), \(I(v + s) = (875v + 875s)I = 1,750\), and \(IIc = 1,750\); \(I(v + s) = IIc\), i.e. ‘there is a functional arrangement […] of the schema’s elements] such that reproduction begins again on the same scale [i.e. simple reproduction].’\(^\text{36}\) (As Marx later\(^\text{37}\) makes explicit, a condition for expanded reproduction in department I, which we shall see is itself a precondition for expanded reproduction per se, is precisely that \(I(v + s) > IIc\).)

Let us assume in case (a) that half the surplus-value in both department I and department II is spent as revenue, i.e. unproductively consumed, and the other half accumulated. Then, if 500\(I\) is accumulated, \((1,000v + 500s)\) is spent as revenue. Hence, we have the exchange \((1,000v + 500s) \leftrightarrow 1,500IIc\). Given too that the 4,000\(I\), as we have already seen,\(^\text{38}\) is disposed of through mutual exchange among the capitalists of department I, all that remains to be considered is the 500\(I\) and the \((376v + 376s)\)\(II\). This will involve considering the movements within the departments, as well as between them.

\(^{33}\) C1, pp. 727, 874.
\(^{34}\) C2, p. 577.
\(^{35}\) This is the first of three numerical schemes of expanded reproduction Marx will present in this chapter; I shall label them, respectively, \(E^1\), \(E^2\) and \(E^3\). Marx will soon abandon \(E^1\), for it does not work. Why it does not work we shall see below.
\(^{36}\) C2, p. 582.
\(^{37}\) C2, pp. 590-1.
\(^{38}\) C2, p. 474.
As we are assuming accumulation of half of \( s \) in department II as well, 188II, has to be transformed into capital, Marx notes that 47II, which he rounds to 48II, has to be transformed into variable capital, and the rest, 140II, into constant capital.\(^{39}\) Now, this 140II, can be transformed into productive capital only insofar as it is replaced by commodities I, to the same value; and these commodities must be means of production, destined either for both departments or exclusively for department II. But 500I, is destined for accumulation in department I, and cannot be exchanged for commodities from II. 140I, is therefore bought unilaterally (i.e. without the money flowing back through subsequent sale) by department II. This, moreover, is repeated each production period. What, Marx asks, is the source of the money for this operation? Marx does not answer this question directly here, but he does reject as a theoretical solution that capitalists could, and even do, pay their workers below the normal wage, directly or indirectly (through the truck system, for example). ‘[I]n an objective analysis of the capitalist mechanism, certain blemishes that still stick to it, and with extraordinary tenacity, cannot be used as subterfuges for getting round theoretical difficulties.’\(^{40}\)

At this point Marx abandons the scheme; why we shall see below.

2 First expanded reproduction scheme

Marx now sets out two complete numerical models of expanded reproduction.\(^{41}\) Before we examine them, it would be useful to remind ourselves of the exchanges necessary to convert the commodity product of departments I and II of one production period into the necessary form to begin the next that we saw under conditions of simple reproduction.

(a) Simple reproduction

We started with:

\[
\begin{align*}
\text{I:} & \quad 4,000_c + 1,000_v + 1,000_s = 6,000 \\
\text{II:} & \quad 2,000_c + 500_v + 500_s = 3,000 \\
\text{total:} & \quad 9,000
\end{align*}
\]

At the end of the production period the following sets of exchanges then took place:

1. 4,000I, – commodity product in the form of means of production – mutually exchanged between capitalists (I);
2. 1,000I, exchanged with 1,000II, and 1,000I, with the other 1,000II,;
3. (500, + 500,)II – means of consumption – bought by workers (II) buying back their own product and capitalists (II) buying back the surplus.

\(^{39}\) Which to my mind appears a slip of the pen, for if the ratio of constant capital to variable capital is 4 : 1 then the portion of 188II, that needs to be converted into variable capital should be one fifth, not one quarter, i.e. 38II, , and that into constant capital therefore not three quarters but four fifths, i.e. 150II, . This is not of great importance, because, for Marx will shortly abandon this scheme.

\(^{40}\) C2 p. 585.

\(^{41}\) Respectively, \(E^2\) and \(E^3\).
These exchanges are represented diagrammatically above. It is important to grasp that the end of one production period, at which point commodity product is sold, is simultaneously the beginning of the next, at which the elements of production and labour-power is bought. The exchange of 4,000$c$, for example, in which the commodity product is realised through what Marx calls mutual exchange in department I, is simultaneously the realisation of the commodity product of one period and the accruing of constant capital for the next: one capitalist’s $C'$–$M'$ is simultaneously another capitalist’s $M$–$C(mp)$.

Disregarding the mediated form of many of the exchanges that occur at this point, we can think of the relation between successive production periods and the general circulation of total social capital like this (the formula for the cycle of productive capital being $P \ldots C'–M–C<_{mp} \ldots P'$):

We assume that workers are paid wages at the beginning of the production period ($M–L$) and that they use this money to buy means of subsistence ($M–C$) produced during it at its end.\(^{42}\)

We can now return to Marx’s first full numerical scheme of expanded reproduction.\(^{43}\)

\(^{42}\) Even though, as Marx noted earlier (C2, p. 156), mass production in fact has only wholesale merchants (and other capitalists) as its immediate purchasers, such that commodities produced do not immediately need to enter into either individual or productive consumption. ‘The consumption of commodities is not included in the circuit of the capital from which they emerge’; i.e. it is only necessary that commodities be sold, not consumed.

\(^{43}\) Under my nomenclature, $E^2$. 
(b) The starting conditions

The starting point for this – the first fully worked-through – model of expanded reproduction is this:

\[
\begin{align*}
\text{I} & : 4,000_c + 1,000_v + 1,000_s = 6,000_{(c + v + s)} \\
\text{II} & : 1,500_c + 750_v + 750_s = 3,000_{(c + v + s)} \\
\text{total:} & : 9,000_{(c + v + s)}
\end{align*}
\]

We should note that now, and necessarily, \(I_{(v + s)} = 2,000 > II_c = 1,500\).

We are maintaining the assumption of no technical change (no change in either organic or value composition), and constant wages (constant rate of surplus-value = 100% in both departments).

Finally, we assume that half of I, is to be accumulated; we make no assumptions as to the rate of accumulation in II.

(c) The exchanges at the end of \(P_t\)

At the start of production period \(P_t\) (which is also the end of \(P_{t-1}\) – one of Marx’s assumptions here of course being already-existing expanded reproduction), capitalists (I) hold, in commodity form, 4,000 constant capital and 1,000 in money to lay out as wages, and capitalists (II) 1,500 in constant capital plus 750 in money for wages.

During the production period, means of production and labour are set to work; with the rate of surplus-value set at 100%, at its end department II holds commodity capital (commodity product in the form of means of production) to the value of \(5,000_{(c + v)} + 1,000_s = 6,000\); and department II \(2,250_{(c + v)} + 750_s = 3,000\) (in the form of means of subsistence). At the end of the period this product needs to be sold, and the money realised converted into factors of production (means of production and labour-power) to begin the next production period (but, this time, on an expanded scale). As the surplus is divided up into distinct parts, we shall adopt the following notation: \(u\) = surplus to be unproductively consumed by capitalists and \(a\) that surplus to be accumulated, and that part of the surplus to be accumulated as constant capital as \(a[c]\) and that as variable capital \(a[v]\) (such that \(s = u + a[c] + a[v]\))

The exchanges that take place are described below, and illustrated in figure 4 below.

- There is an exchange \((1,000_v + 500_u)\)I \(\leftrightarrow 1,500II_c\) \((II_v + \frac{1}{2}I_s) \leftrightarrow II_c\), effectively composed of (a) \(1,000I_v \leftrightarrow 1,000II_c\), workers (I) using wages laid out at the beginning of the period to buy 1,000 means of consumption, and capitalists (II) using this money to buy means of production;\(^{44}\) and (b) \(500I_u \leftrightarrow 500II_c\), capitalists (II) advancing money to purchase means of production and capitalists (I) anticipating the sale of means of production to buy means of consumption, the money flowing back and forth to complete the exchanges.\(^{45}\) This set of exchanges is labelled 1 in the figure 4.

There now remains in commodity product form in I \(4,000_{c} + 500_{u}\), and in II \(750_{v} + 750_{s}\).

- The conversion of 4,000I, from means of production, into money, and then into means of production again, is achieved, exactly as we have seen under conditions of simple reproduction, through mutual exchange among capitalists (I).\(^{46}\) This is labelled 2.

The commodity product that remains now in I is \(500_{u}\) and in II \(750_{v} + 750_{s}\).

\(^{44}\) Cf. C2, p. 571.

\(^{45}\) As we saw in C2, pp. 475-7.

\(^{46}\) C2, pp. 498-501.
We assume the division of 500I\textsubscript{a} into 400I\textsubscript{a}[c] and 100I\textsubscript{a}[v] (maintaining the ratio of \(c : v\) in department I as 4 : 1).

- The conversion of 400I\textsubscript{a}[c] from its natural form of means of production, into money, and then again into the natural form of constant capital occurs among the capitalists of department I, as we saw above\(^47\) (and as the 4,000I\textsubscript{c} is converted). Compared with simple reproduction, what has happened here is that a part of the surplus product of department I, rather than being exchanged with means of subsistence, is bought and sold among the capitalists of department I, the value staying within the department. This is why \(I_{(v + s)} = 2,000\) \textit{must} > \(II_{c} = 1,500\). This is labelled 3.

There now remains (as commodity product): in I, 100I\textsubscript{a}[v] ; and in II, 750\(v+750\).s.

- The 100I\textsubscript{a}[v], in the form of means of production, is bought by department II as extra constant capital with money advanced in anticipation of the realisation of the sale of 100\(II_{s}\) means of consumption, money which flows back when the sale is realised. There is a problem here, however. This money will function as variable capital for department I: the workers to whom this value will pass, in the form of wages, are ‘extra’ workers, necessary for the expanded scale of production to be carried out in the next production period. This 100\(II_{s}\), therefore exists at the end of \(P_t\) in the form of commodity \textit{product}, not money – the product can only be realised

\(^{47}\) C2, pp. 572-7.
in the next production period. Under the operating assumptions here, the sale of this part department II’s (surplus) product is forestalled until the next production period. This notwithstanding, we are the moment going to make the following assumption. Earlier in the volume, Marx noted that, under conditions of mass production, capitalists do not sell directly to workers (or other capitalists), but to wholesale merchants, who then sell the product on, directly or indirectly, to whoever will (productively or unproductively) consume it. The commodity product thus does not necessarily have to enter into either individual or productive consumption immediately. ‘The consumption of commodities is not included in the circuit of the capital from which they emerge’; it is only necessary that commodities be sold, not consumed. Here, we shall assume that they are sold at the end of the production period, and that the money advanced by department II returns to it (even if from outside the circulation movement we are considering here). 4 above.

Remaining commodity product: II(750\(_v\) + 650\(_s\)).

- The commodity product 750II\(_v\) is bought back by workers (II) with their wages. This is labelled 5.

650II\(_s\) remains.

- As department II has bought 100I\(_s\) in the form of means of production (in 4 above), an extra 50\(_a[v]\) needs to be found for the next production period to put this means of production to work (maintaining the value composition \(c : v\) as 2 : 1 in department II). This comes from what would otherwise be the capitalists’ consumption fund II\(_s\) : rather than consisting of means of consumption to be exchanged among and then consumed unproductively by capitalists, this part of the surplus product exists in the form of means of subsistence to be consumed by workers. However, again, these workers are ‘extra’ workers, necessary for the expanded scale of production to be carried out in the next production period; therefore the demand to buy this product only exists when the extra variable capital is paid to these workers in the form of wages. Again, however, we shall make the assumption (as we did in the case of 4 above that the commodity is sold (if not consumed) at the end of the production period, and that money is advanced (by merchant capitalists). This exchange is labelled 6.

Remaining commodity product: 600II\(_s\).

- Because of the accumulation of 150II\(_s\), the capitalist consumption fund now stands at 600; 600II\(_u\) is realised through exchange among the capitalists of department II. This is labelled 7.

(d) The distribution of the product at the end of \(P_t\)

At the end of the period (i.e. at the beginning of the next one) the ‘collective capitalists’ of each department are in possession of this:

---

48 Note Marx’s comment above (C2, p. 580) regarding the formation of a ‘commodity stock in means of consumption that ensures the continuity of the consumption process involved in reproduction, and therefore the transition from one year to the other.’

49 C2, p. 156.

50 A ‘further 50\(_v\) has to be added for the purchase of new labour-power for working this up, and so its variable capital grows from 750 to 800. This extension of both department II’s constant and variable capital is met out of its own surplus-value; of the 750 II\(_s\), therefore, there only remains 600\(_s\) as a consumption fund for the department II capitalists [...]’ C3, p. 586.
Commodity product and money held by capitalists at the start of $P_{t+1}$

<table>
<thead>
<tr>
<th></th>
<th>total commodity product produced in $P_t$</th>
<th>constant capital (in commodity form)</th>
<th>variable capital (money)</th>
<th>unaccumulated surplus (in commodity form; to be consumed unproductively)</th>
</tr>
</thead>
<tbody>
<tr>
<td>department I</td>
<td>6,000</td>
<td>4,400</td>
<td>1,100</td>
<td>500</td>
</tr>
<tr>
<td>department II</td>
<td>3,000</td>
<td>1,600</td>
<td>800</td>
<td>600</td>
</tr>
</tbody>
</table>

The total capital (constant plus variable) at the end of the period (to be deployed at the start of the next one, $P_{t+1}$) is therefore $5,500I + 2,400II = 7,900$, whereas at the beginning of the period we had $5,000I + 2,250II = 7,250$.

(e) Accumulation during $P_{t+1}$

If we now assume that production proceeds on this expanded basis, at the end of $P_{t+1}$ we will have, maintaining the rate of surplus-value at 100% the following commodity product:

- **department I:** $4,400_c + 1,100_v + 1,100_s = 6,600$
- **department II:** $1,600_c + 800_v + 800_s = 3,200$

If we further assume that in department I accumulation continues at the same rate, i.e. that one half of the surplus-value, i.e. $550I_u$, is accumulated, then we see immediately that we have a problem with the exchange $[I_v + \frac{1}{2}I_u] \leftrightarrow II_c$, for while $[I_v + \frac{1}{2}I_u] = 1,650$, $II_c = 1,600$. All of the variable capital laid out in department I passes to department II (in payment for workers’ means of subsistence); department II then uses this money to buy means of production back from department I. After this has happened, the surplus commodity product remaining in department I (i.e. in form of means of production) that is not to be accumulated has to be exchanged with department II; has to be converted into means of subsistence – i.e. has to be converted into means of production for department II (otherwise it cannot be realised). In short, the shortfall between the commodity product in department II equivalent to the outlay on constant capital on the one hand and the commodity product in department I equivalent to the capital laid out by department I in the form of variable capital, has also to be accumulated in department II as constant capital (in addition to the commodity product equivalent to the capital to be laid out by department I in the form of variable capital).

This is the sense in which Marx makes the (otherwise cryptic) comment that ‘[i]f things are to proceed normally, accumulation in department II must take place quicker than in department I, since the part of $I_v + I_u$ that has to be exchanged for commodities $II_c$ would otherwise grow more quickly than $II_c$, which is all that it can be exchanged for.’

In other words, the rate of accumulation in department II is forced upon it by the need to soak up that part of department I’s commodity product that is not going to be accumulated as constant capital in department I itself.

Unlike the case of the constant capital accumulated in department II as a consequence of the variable capital accumulated in department I, here there is now no realisation problem. The commodity product in department II equivalent to the constant capital accumulated is realised by department I capitalists immediately realising the

---

51 C2, p. 588.
purchase of means of capitalists’ subsistence (either returning the money advanced by department II, or anticipating
its return to them).

But in addition, now, to maintain department II’s ratio of \( c : v \) of 2 : 1 a further 25\( \Pi_2 \), also needs to be deployed as
variable capital. This value, existing in the form of means of subsistence, is now destined for workers’ rather than
capitalists’ consumption: to be realised through workers’ wages rather than through mutual exchange amongst
capitalists. Again, we shall make the assumption that this part of department II’s commodity is successfully realised
(by sale to merchants) without worrying too much about exactly how.

If we assume that the ratio \( c : v \) in department I is maintained, then of the 550\( \Pi_1 \) to be accumulated 440 is so as constant capital and 110 as variable; this latter, 110\( \Pi_1 \), is realised against \( \Pi_2 \) in the form of means of production to be accumulated in department II; this means that means of consumption to the value of 110 are consumed by workers (I) rather than by capitalists (II) – the latter, rather than consuming this surplus are now ‘forced’ to capitalise it (as means of production, i.e. as constant capital); but alongside this, because of \( c : v = 2 : 1 \), a further 55\( \Pi_1 \), has to be capitalised as variable capital. (And it is again the case that this value, as commodity product, could only be realised in the next production period, a problem we shall again circumvent by assuming sale to merchant capitalists.)

In summary, then, of the 800\( \Pi_2 \) with which we started \( P_{t+1} \), we have subtracted 50 to be deployed as constant
capital, so as to facilitate \( [\Pi_1 + \frac{1}{2} \Pi_u] \leftrightarrow \Pi_c \), and 25, along with this, as additional variable capital; and now a further
110, along with another 55, to be capitalised as constant and variable capital respectively; all this leaving a capitalist
consumption fund in department II of 560\( \Pi_u \).

(f) The exchanges at the end of \( P_{t+1} \)
At the start of \( P_{t+1} \), then, capitalists (I) hold, in commodity form, 4,400 constant capital and 1,100 in money to lay
out as wages, and capitalists (II) 1,600 in constant capital plus 800 in money for wages. Assuming a rate of surplus-
value of 100 \%, at its end of this period department I holds commodity capital (commodity product in the form of
means of production) to the value of \( 5,500(c + v) + 1,100_s = 6,600 \); and department II \( 2,400(c + v) + 800_s = 3,200 \)
in the form of means of subsistence), product which needs to be sold, and the money realised converted into
factors of production (means of production and labour-power) to begin the next production period (on an even
greater expanded scale). These are the exchanges that take place.

1 [\( \Pi_v + \Pi_u \) \( \leftrightarrow \Pi_c \). This is composed of (a) 1,100\( \Pi_v \leftrightarrow 1,100\Pi_c \); and (b) 500\( \Pi_u \leftrightarrow 500\Pi_c \).

2 For the reasons explained above, after this exchange 50\( \Pi_u \) (that part of the surplus not to be accumulated but
yet disposed of) remains. This 50\( \Pi_u \), in the form of means of production, is bought by department II as extra
constant capital with money advanced in anticipation of the realisation of the sale of 50\( \Pi_2 \) means of

\[52\] Marx’s word, my italicisation: C2, p. 588. Marx ‘shows clearly that the accumulation by Department II is completely
determined and dominated by the accumulation of Department I. [...] Though there is nothing to choose between the
capitalists of the two departments as far as the figures are concerned, it is quite obvious that Department I has taken the
initiative and actively carries out the whole process of accumulation, while Department II is merely a passive appendage. This
dependence is also expressed in the following precise rule: accumulation must proceed simultaneously in both departments,
and it can do so only on condition that the provisions-department increases its constant capital by the precise amount by
which the capitalists of the means-of-production-department increase both their variable capital and their fund for personal
criticism of Marx’s exposition, this should be read as expressing a fundamental fact about the patterns of accumulation in
capitalist reproduction.
consumption; we assume again here the sale of this last to merchant capitalists; the money advanced to department II will function as variable capital in the next period.

3 This 50$I_a$, which passes to department II as means of production, needs another 25$I_d[v]$ to set it to work. We again assume the sale of the commodity product and the advance of money (to function as variable capital).

4 Of the 550$I_a$, 440$I_a[c]$ is realised through mutual exchange within department I.

5 The remaining 110$I_d[v]$ is bought by department II with money advanced in anticipation of the sale of the commodity product (which we assume) and accumulated as constant capital in department II.

6 This constant capital to be accumulated in II needs 55 variable capital. This is advanced on the anticipation of the here assumed sale of the commodity product.

7 The 4,400$I_c$ is realised through mutual exchange among the capitalists of department I.

8 The 800$I_v$ is bought back by the workers of department II with their wages.

9 The remaining 560$I_s$ is realised through mutual exchange among the capitalists of department II to be unproductively consumed.

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**Figure 5**: scheme $E^2$: the exchanges at the end of $P_{t+1}$
At the end of the production period (i.e. at the beginning of the next one), this is what we have:

<table>
<thead>
<tr>
<th>Department</th>
<th>Total Commodity Product Produced in $P_{t+1}$</th>
<th>Constant Capital (in commodity form)</th>
<th>Variable Capital (money)</th>
<th>Unaccumulated Surplus (in commodity form; to be consumed unproductively)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Department I</td>
<td>6,600</td>
<td>4,840</td>
<td>1,210</td>
<td>550</td>
</tr>
<tr>
<td>Department II</td>
<td>3,200</td>
<td>1,760</td>
<td>880</td>
<td>560</td>
</tr>
</tbody>
</table>

(g) *Accumulation during $P_{t+2}$ and beyond*

We assume now that accumulation proceeds on this same basis, with half of department I’s surplus-value being accumulated. Accumulation in department II is driven by this: the $I_s$ to be converted into variable capital in department I is bought by department II in the form of means of production as extra constant capital, and one half of this value (to maintain the ratio between constant and variable capital in the department) is also accumulated by II, out of its surplus-value, as variable capital. In addition, the shortfall on the part of II in the exchange $[I_v + \frac{1}{2}I_s] \leftrightarrow II_s$ is supplemented with an equivalent $II_s$, accumulated as constant capital, in turn supplemented by half this value, to be accumulated as variable capital. A table of values for the numerical development of $E^2$ is given below.

Table 1: Scheme $E^2$ (Marx’s starting condition figures in red)
2 Second expanded reproduction scheme

We start with the following conditions.

\[
\begin{align*}
\text{I} & \quad 5,000_c + 1,000_v + 1,000_s = 7,000 \\
\text{II} & \quad 1,430_c + 285_v + 285_s = 2,000 \\
\text{total} & \quad = 9,000
\end{align*}
\]

The ratio \(c:v\) in both departments is 5:1.\(^{54}\) We assume a rate of accumulation of surplus value of 50%.

Hence, again, in the exchange \([ I_v + \frac{1}{2} I_s] \leftrightarrow \text{II}\), there is a shortfall, of 70, which is provided from \(\text{II}_s\) (which, after this deduction, stands at \(215\ \text{II}_s\)). In addition to this \(70\text{II}_s\), to be accumulated as constant capital, \(14\text{II}_s\) needs to be accumulated as variable capital. \(\text{II}_s\) now stands at \(201\text{II}_s\).

Of the \(500\_s\) to be accumulated, \(417\) will be so as constant capital and \(83\) as variable (\(c:v = 5:1\)). In addition, as in the previous example, the \(I_s\) to be accumulated as variable capital (\(83\_s\)) withdraws a further equivalent value from \(\text{II}_s\), which is accumulated as constant capital, which in turn withdraws an additional \(17\text{II}_s\), to be accumulated as variable capital.

As in the first example, we shall assume the sale of the commodity product of department II equivalent to the variable capital accumulated in both departments.

Proceeding thus, for period \(P_t\), we have the following product and its distribution:

**department I**

\[
\begin{align*}
\text{product:} & \quad 5,000_c + 1,000_v + 1,000_s = 7,000 \\
\text{distribution:} & \quad [5,000_c + 417_s = 5,417]_c + [1,000_v + 83_s = 1,083]_v + 500_s^* = 7,000 \\
& \quad \text{capitalists’ consumption fund}
\end{align*}
\]

**department II**

\[
\begin{align*}
\text{product:} & \quad 1,430_c + 285_v + 285_s = 2,000 \\
\text{distribution:} & \quad [1,430_c + 70_s + 83_s = 1,583]_c + [285_v + 14_s + 17_s = 316_s] + 101_s^* = 2,000 \\
& \quad \text{capitalists’ consumption fund}
\end{align*}
\]

The exchanges that take place at the end of \(P_t\) are illustrated in figure 6 below.

Production period \(P_{t+1}\) now proceeds like this:

\[
\begin{align*}
\text{I:} & \quad 5,417_c + 1,083_v + 1,083_s = 7,583 \\
\text{II:} & \quad 1,583_c + 316_v + 316_s = 2,215
\end{align*}
\]

Reproduction continues as it did with regard to \(E^2\). The numerical development of the scheme is shown in table 2.

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\(^{53}\) E^3.

\(^{54}\) A fact, Marx observes, which ‘presupposes a significant development of capitalist production and, accordingly, of the productivity of social labour as well; a significant prior expansion of the scale of production; and […] a development of all the circumstances that produce in the working class a relative surplus population.’ C2, p. 589.
### Table 2: Scheme $E^3$ (Marx’s starting condition figures in red)

<table>
<thead>
<tr>
<th>Production Period</th>
<th>Department I</th>
<th>Department II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>c</td>
<td>v</td>
</tr>
<tr>
<td>t</td>
<td>5000.0</td>
<td>1000.0</td>
</tr>
<tr>
<td>t+1</td>
<td>5416.7</td>
<td>1083.3</td>
</tr>
<tr>
<td>t+2</td>
<td>5868.1</td>
<td>1173.6</td>
</tr>
<tr>
<td>t+3</td>
<td>6377.1</td>
<td>1271.4</td>
</tr>
<tr>
<td>t+4</td>
<td>6886.8</td>
<td>1377.4</td>
</tr>
<tr>
<td>t+5</td>
<td>7460.7</td>
<td>1492.1</td>
</tr>
<tr>
<td>t+6</td>
<td>8082.4</td>
<td>1616.5</td>
</tr>
<tr>
<td>t+7</td>
<td>8756.0</td>
<td>1751.2</td>
</tr>
<tr>
<td>t+8</td>
<td>9485.6</td>
<td>1897.1</td>
</tr>
<tr>
<td>t+9</td>
<td>10276.1</td>
<td>2055.2</td>
</tr>
<tr>
<td>t+10</td>
<td>11132.5</td>
<td>2226.5</td>
</tr>
</tbody>
</table>

**Figure 6:** Scheme $E^3$: the exchanges at the end of $P_t$
3 The limits to accumulation

(a) General considerations

Under conditions of simple reproduction, by definition, all the surplus value is consumed unproductively. The commodity product of department I is sold to capitalists of department I and to capitalists of department II, while all of department II’s product is sold to workers and capitalists of department I and department II. If we assume that the constant capital that department I needs is unproblematically exchanged within department I, and that the means of subsistence consumed within department II – both by workers and capitalists – are unproblematically cleared within the department, the key question is that of the balance of exchanges between the departments, which are evidently composed of (1) the constant capital that department II needs from department I and (2) the means of subsistence that department I needs from department II. If (1) = IIc (to replace the means of production necessary for continued production) and (2) = I(v+s) (the means of subsistence the workers and capitalists need to consume), the condition for balanced reproduction is that IIc = I(v+s). The mechanism of the exchange is that the capitalists of department II buy means of production to the value of their commodity product IIc, while the capitalists of department I buy means of subsistence to the value of the surplus product I, and the workers of department I spend their wages (Iv) of means of subsistence too. The money to the value of Iv received by the capitalists of department I is used as variable capital at the beginning of the next production period.

In the case of expanded reproduction it is still the case that all of department I’s commodity product needs to be sold to the capitalists of department I and department II in the form of means of production, and all of department II’s – means of workers’ and capitalists’ subsistence – to the workers and capitalists of department I and department II. If we again assume that the reproduction of department I’s constant capital takes place through mutual exchange among the capitalists of the department, and that the reproduction of department II’s variable capital also takes place within the department – in this case through workers buying back the commodity product with their wages – then the key question with regard to whether or not balanced reproduction occurs – and by ‘balanced reproduction’ what is meant here is that, within the assumptions of the model, all commodity product is realised such that expanded reproduction will occur in the next production period – is the exchanges through which commodity product components IIc, IIv, Ic and Iv are realised.

Under conditions of expanded reproduction, the means of subsistence that department I needs to buy from department II must be equal to the capital laid out in department I on wages, plus that part of the surplus-value to be accumulated as new variable capital, plus that part of the surplus-value to be unproductively consumed by the capitalists. Department II, on the other hand, needs to buy from department I the constant capital to maintain simple reproduction (= IIc) and the constant capital equivalent to that part of the surplus product of department II to be accumulated as such (and we assume that the product of each department consumed within the department is unproblematically exchanged).

Therefore, denoting the value product of each department $c + v + u + a$, where $u$ = surplus to be unproductively consumed by capitalists and $a$ that surplus to be accumulated, and that part of the surplus to be accumulated as constant capital as $a[c]$ and that as variable capital $a[v]$ (such that $s = u + a[c] + a[v]$), then the product of each department will be:

\[
\begin{align*}
I &: Ic + Iv + Iu + Ia[\text{c}] + Ia[\text{v}] \\
II &: IIc + IIv + IIu + IIa[\text{c}] + IIa[\text{v}]
\end{align*}
\]

The exchanges between the departments that take are, again, that between (1) the constant capital that department II needs from department I and (2) the means of subsistence that department I needs from department II. (1) evidently = IIc + IIa[\text{c}], and (2) = Ic + Iv + Ia[\text{v}]. Hence the equality for balanced reproduction needs to be:

\[
IIc + IIa[\text{c}] = Ic + Iv + Ia[\text{v}]
\]
This is the ‘Bukharin formula’ condition for expanded reproduction.\(^{55}\) Put into plain words, what it says is that, in a given production period, the capital laid out on constant capital by department II plus the surplus-value to be accumulated by department II as constant capital in the next production period must equal the capital laid out in wages in department I plus the surplus to be accumulated as variable capital plus the surplus to be unproductively consumed by the capitalists.

There is an immediate and important conclusion to be drawn from this. \(\Pi_v + \Pi_u\) is the demand on the part of department II for means of production, for reproduced and accumulated constant capital; \(I_v + I_u + I_d[v]\) the commodity product it receives. If \(I_v = I_u + I_d[c] + I_d[v]\), then \(I_u + I_d[v] = I_v - I_d[c]\), so we can rewrite the condition as \(\Pi_v + \Pi_d[c] = I_v + I_d - I_d[c]\). If \(I_d[c] > 0\), then \(\Pi_v + \Pi_d[c] < I_v + I_d\), in other words that the new value created in the production of means of production must be greater than the demand for means of production of means of subsistence. This is the sense in which Marx makes this remark:

> The distinction [between simple and expanded reproduction] lies only in the form of the surplus labour applied, the concrete character of its particular useful mode. It has been spent on [...] means of production for means of production instead of on means of production for means of consumption. In the case of simple reproduction, it was assumed that the whole of the surplus-value in department I was spent as revenue, i.e. on commodities from department II; it consisted only of those means of production needed to replace the constant capital \(I_c\) in its natural form. Thus in order to make the transition from simple reproduction to expanded reproduction, production in department I must be in a position to produce fewer elements of constant capital for department II, but all the more for department I.\(^{56}\)

There is no similar necessary relationship between the demand on the part of department I for the commodity product produced in department II and the new value created in the latter, between \(I_v + I_u\) on the one hand and \(\Pi_v + \Pi_u\) on the other. The relationship between the departments in the exchange \(I_v + I_u\) is not symmetrical.

But if the condition for balanced extended reproduction is \(\Pi_v + \Pi_u\) = \(I_v + I_u + I_d[v]\), what does this tell us about the relationships between the pattern of accumulation in one department and that in the other? To what extent does the rate of accumulation in department I affect that in department II, and vice versa?

The exchange \(I_v + I_u\) is composed of the two components: \(I_v\), in which the workers of department I use their wages to buy means of subsistence, money which flows back as the capitalists of department II use it to buy means of production; and \(I_u\), in which the reproduction of department II’s constant capital is completed (the assumption that \(I_v > I_u\) being the only realistic one given a certain minimum stage of capitalist development). The surplus realised here by department I – in its realised form means of subsistence – is necessarily consumed unproductively by capitalists (I). Hence the exchange \(I_v + I_u\) is composed of the two components \(I_v\) and \(I_u\). It may be the case that \(I_v + I_u > I_v\) (as is the case in period \(P_{t+1}\) in scheme \(E^2\) above, and illustrated in figure 6 above); if so, the excess, the value \(I_v + I_u - I_v\), is necessarily accumulated in department II as constant capital. If \(I_v + I_u < I_v\) (because of a high rate of accumulation in department I), then the excess \(I_v\) is realised through money advanced (to buy means of production) in the anticipation of it flowing back after passing back having passed through the form of variable capital (workers’ wages). This excess is therefore accumulated in department I as variable capital (this is illustrated in figure 7 below).

\(^{55}\) Kenneth J Tarbuck (ed.), ‘The Accumulation of Capital – An Anti-Critique’ by Rosa Luxemburg and ‘Imperialism and the Accumulation of Capital’ by Nikolai Bukharin (New York and London, 1972), p. 158, where the formula appears as \((v_1 + B_{1r} + \alpha_1) = (c_2 + B_{2r})\)

\(^{56}\) C2, p. 572
What remains in department I is consumed productively, i.e. accumulated, as constant and variable capital in function of the existing composition of capital (which here we are assuming constant): \( I_a = I_{a[c]} + I_{a[v]} \). If the value composition of capital, \( \alpha, = \frac{c}{c + v} \), then we have \( I_a = \alpha_I I_a + (1 - \alpha_I) I_a \).

But \( I_a \) also = \( r_I I_I \), where \( r \) = the rate of accumulation. So, in addition, \( I_a = \alpha_I r_I I_I + (1 - \alpha_I) I_I \).

For the value accumulated as variable capital in department I an equivalent value is necessarily accumulated as constant capital: that part of the surplus product destined to be accumulated as variable capital in department I is realised by being bought by department II with money advanced on the strength of it flowing back as the workers of department I use their wages to buy means of subsistence. But this is not necessarily the only constant capital accumulated in department II: as we have seen, if \( I_v + I_u > II_c \), the excess, the value \( I_v + I_u - II_c \), is also accumulated as constant capital in department II too.

Finally, if \( \alpha = \frac{c}{c + v} \), for a given value \( c, v = c \left( \frac{1 - \alpha}{\alpha} \right) \), while \( c = v \left( \frac{\alpha}{1 - \alpha} \right) \).

We can now start to look at how the pattern of accumulation in one department affects that of the other. We shall start, as Marx has done, by looking at how what happens in department I affects what happens in department II.

**(b) Department I determining**

The surplus accumulated in department I is the sum of the following four elements:

1. Value equivalent to the surplus-value accumulated as variable capital in department I (accumulated as constant capital in department II), i.e. \( I_{a[v]} = r_I I_I (1 - \alpha_I) \).

2. The shortfall in the exchange \( I(v + j) \leftrightarrow II(c) \) (if there is one), accumulated as constant capital, given by \( I_v + I_u - II_c \). But since \( I_u = I_x(1 - r_I) \), here we have \( I_v + I_x(1 - r_I) - II_c \).

3. Taking into account the value composition of department II, the surplus-value to be accumulated as the variable capital necessary to set constant capital (1) in motion: \( I_{a[v]} \left( \frac{1 - \alpha_{II}}{\alpha_{II}} \right) \), i.e. \( r_I I_I (1 - \alpha_I) \left( \frac{1 - \alpha_{II}}{\alpha_{II}} \right) \).

4. Again taking into account the value composition, the variable capital necessary to set constant capital (2) in motion, hence \( I_v + I_x(1 - r_I) - II_c \left( \frac{1 - \alpha_{II}}{\alpha_{II}} \right) \).

The total constant capital to be accumulated in department II is therefore

\[
II_{a[c]} = \left[ r_I I_I (1 - \alpha_I) \right] + \left[ I_v + I_x(1 - r_I) - II_c \right] \text{ (which simplifies to } II_{a[c]} = I_v + I_x - II_c - r_I I_I \alpha_I \text{)}
\]

and the total accumulated surplus

\[
\begin{align*}
II_a &= \left\{ I_v + I_x - II_c - r_I I_I \alpha_I \right\} + \left\{ I_v + I_x - II_c - r_I I_I \alpha_I \left( \frac{1 - \alpha_{II}}{\alpha_{II}} \right) \right\} \\
\end{align*}
\]

57 ‘Differing organic compositions of capitals are thus independent of their absolute magnitudes. The only question is always how much of each 100 units is variable capital and how much is constant.’ Karl Marx, *Capital*, vol. 3 (Harmondsworth, 1981), p. 248.
Here, of course, it is indeed true that ‘Department I has taken the initiative and actively carries out the whole process of accumulation, while Department II is merely a passive appendage.’ But is there a limit beyond which department I cannot accumulate?

The equation II_{a[c]} = [\eta \cdot I_r \cdot (1 - \alpha)] + [I_r + I_r \cdot (1 - r_l) - II_c] shows us that the constant capital accumulated in department II comes from two sources: the variable capital accumulated in department I \( (\eta \cdot I_r \cdot [1 - \alpha]) \), and the excess in the exchange \( I_{(p + g)} \leftrightarrow II_c \cdot (I_r + I_r \cdot [1 - r_l] - II_c) \). If the latter is negative, i.e. \( I_r + I_r \cdot (1 - r_l) < II_c \), then that equivalent part of the variable capital accumulated in department I does not cause an equivalent accumulation of constant capital in department II, impeding its overall accumulation (in addition, we see again the problem that this part of the commodity product \( II_c \) realised in exchange against \( I_{a[v]} \) will only be realised as the workers (I) spend their wages in the next production period). Nevertheless, if the magnitude of \( I_r + I_r \cdot (1 - r_l) - II_c \) is less that part accumulated in the realisation of that part of II, remaining after the exchange \( I_{(p + g)} \leftrightarrow II_c \), then accumulation will still occur in department II. This case is illustrated – under the starting conditions of \( E_2 \), setting the rate of accumulation in department I at 0.6 (such that \( I_r + I_r \cdot (1 - r_l) = 1,400 \)) – in figure 7 below.\(^{58}\)

Looking again at the equation II_{a[c]} = [\eta \cdot I_r \cdot (1 - \alpha)] + [I_r + I_r \cdot (1 - r_l) - II_c], under what conditions will the effect on the constant capital accumulated in department II of the excess of II, against I_r, not be compensated for by additional accumulation of variable capital in department I? Evidently, when \( \eta \cdot I_r \cdot (1 - \alpha) + [I_r + I_r \cdot (1 - r_l) - II_c] = 0 \) (in which case II_{a[c]} and hence II_{a[v]} = 0 too). Rewriting \( \eta \cdot I_r \cdot (1 - \alpha) + [I_r + I_r \cdot (1 - r_l) - II_c] = 0 \), we arrive at \( \eta = \frac{I_r + I_r \cdot (1 - r_l) - II_c}{I_r \cdot \alpha} \). This value, 0.625 in the case of the starting conditions of \( E_2 \), is the limit at which, while accumulation takes place in department I, department II can only continue production in the next period on the same scale. The

---

\(^{58}\) No particular importance should be attached to the use of \( E_2 \) rather than \( E_3 \) in this and following illustrations. It was chosen because it is the scheme worked out in more detail and with slightly more manageable numbers; \( E_3 \) as an illustration would depict the same mechanisms exactly, however.
reason for this is that at this point all of the variable capital to be accumulated in department I is now accumulated through the exchange with II, in other words, $I_v - (I_v + I_u) = I_{a[v]}$. The exchanges that take place at the end of the first period with this value of $r_1$ are illustrated in figure 8 below.

If the rate of accumulation rises above this level, i.e. if $r_1 > \frac{I_v + I_u - II_v}{I_v \cdot \alpha_I}$, then there is absolute overaccumulation in department I (‘absolute’ because the overaccumulation in department I is independent of the conditions in department II); although accumulation continues here, department II can no longer sell all of its commodity product. What has happened here is that now $I_v + I_u + I_{d[v]} > II_v$. Department II cannot realise all of its commodity product $II_v$, and cannot reproduce its constant capital. With $r_1 = 0.7$, for example, department the exchanges that take place at the end of the first production period (maintaining the starting conditions as above) are illustrated in figure 9 (unsold commodity product indicated in red).

(b) Department II determining

So far here we have taken as the independent variable the rate of accumulation in department I, and identified the limit beyond which department I overaccumulates. But we can also take as the independent variable the rate of accumulation in department II.

At the end of any given production period, the capitalists of department II need to reproduce their constant capital and accumulate more constant capital in function of the rate of accumulation. This, in the form of means of production, to the value of $II_v + II_{d[c]}$, they buy from department I in the exchange $II_v \leftrightarrow I_{(v+s)}$. In the component $II_v \leftrightarrow I_v$ of this exchange, workers in department I buy means of subsistence with their wages from department II, while department II uses the money to buy means of production. In the $II_v \leftrightarrow I_s$ component, capitalists (II) advance money (on the strength of it returning in the purchase of means of subsistence, either by capitalists, or by workers) to
complete the purchase of means of production. At the end of all this, capitalists (II) hold means of production to the value of II $c + II \alpha \alpha \alpha$. capitalists (I) a money reserve to the same value, plus the rest of their surplus product.

This last now cannot be consumed unproductively – it cannot be transformed into means of subsistence, for department II now has no more need of means of production: it has to be productively consumed, i.e. accumulated as constant capital. But this constant capital value needs labour-power to set it to work: variable capital, in function of the value composition of capital, is advanced from the money reserve held by capitalists (I); the remainder of the reserve is used to buy means of subsistence which the capitalists then unproductively consume.

The constant capital accumulated by department II is given by $II \alpha = r_{II} II \alpha \alpha \alpha$. The money reserve held by department I after the exchange $II \leftrightarrow I \alpha \alpha \alpha$ is equal to this value less $I \alpha \alpha \alpha$, hence $II \alpha + r_{II} II \alpha \alpha \alpha - I \alpha \alpha \alpha$; the surplus commodity product in form of means of production left in department I is what is left in $I \alpha \alpha \alpha$ after the subtraction of this value, therefore $I \alpha - (II \alpha + r_{II} II \alpha \alpha \alpha - I \alpha \alpha \alpha)$, or $I \alpha \alpha \alpha - II \alpha - r_{II} II \alpha \alpha \alpha$. This is $I \alpha \alpha \alpha$, what is accumulated in department I as constant capital.

What is accumulated as variable capital is hence $(I \alpha \alpha \alpha - II \alpha - r_{II} II \alpha \alpha \alpha)$, and the capitalists’ unproductive accumulation fund is the difference between this value and the money reserve it holds:

$$(II \alpha + r_{II} II \alpha \alpha \alpha - I \alpha \alpha \alpha) - [(I \alpha \alpha \alpha - II \alpha - r_{II} II \alpha \alpha \alpha + I \alpha \alpha \alpha) \left(\frac{1 - x_{II}}{x_{I}} \right)]$$.

Evidently, however, if the demand for constant capital remaining on the part of department II after the exchange with $I \alpha \alpha \alpha$ is greater than $I \alpha \alpha \alpha$ then the reproduction/accumulation of department II’s capital will not be completed. Hence, for accumulation in both departments to occur, $II \alpha + r_{II} II \alpha \alpha \alpha - I \alpha \alpha \alpha < I \alpha \alpha \alpha$. This gives a value for $r_{II} <$
\[
\frac{I_v + I_s - II_c}{II_s.\alpha_{II}}; \text{ when } r_{II} = \frac{I_v + I_s - II_c}{II_s.\alpha_{II}} \text{ department II will accumulate while in department I production will only be continued on the same scale; when } r_{II} \text{ rises above this value its own reproduction is disrupted.}
\]

Imagine the following starting conditions:

I \hspace{1cm} 4,000_c + 1,000_v + 1,000_s \\
II \hspace{1cm} 1,800_c + 450_v + 450_s \\

\alpha_I = \alpha_{II} = 0.8. \text{ Imagine now that department II now accumulates at a rate of 0.6. The constant capital it is to accumulate } = r_{II}.II_c.\alpha_{II} = 216 \text{ (with } II_{d[v]} = 54). \text{ (The limit } r_{II} = \frac{I_v + I_s - II_c}{II_s.\alpha_{II}} = 0.55; \text{ } II_s.\alpha_{II} + I_v = 1,016; \text{ and } I_v = 1,000. \text{ There is therefore a shortfall of } 16II_{d[c]}, \text{ held by department II as commodity product which it cannot convert into means of production to accumulate as constant capital (and an equivalent } 4II_{d[v]}). \text{ Department II is thus forced to reduce its rate of accumulation to a level at which department I would be able to apportion it the means of production necessary to fulfill it. The exchanges that occur at } r_{II} = 0.6 \text{ are illustrated in figure 10 below (unsold commodity product in red).}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure10.png}
\caption{absolute overaccumulation in department II}
\end{figure}

\textbf{(c) Both departments (and therefore neither) determining}

It is clear, therefore, that the rate of accumulation in one department has consequences for how accumulation takes place in the other. But it is also clear that setting the rate of accumulation of one department or the other as the independent variable in determining patterns of accumulation in both is one-sided; in reality, both rates of accumulation are determined independently of each other (or are determined by common factors independent of the rate of accumulation operative in the other department).
If we say that the constant capital accumulated in department I is determined in function of the rate of accumulation in department II, then the former is given by $I_a^{[c]} = I_v + I_s - II_c - r_{II}II_s\alpha_{II}$. But, by definition, it is also true that $I_a^{[c]} = r_I I_s\alpha_I$. Hence, if reproduction is to occur, $r_I I_s\alpha_I = I_v + I_s - II_c - r_{II}II_s\alpha_{II}$. This in turn gives us two expressions for the value of one rate of accumulation in terms of the other.

$$r_I = \frac{I_v + I_s - II_c - r_{II}II_s\alpha_{II}}{I_s\alpha_I}$$

and

$$r_{II} = \frac{I_v + I_s - II_c - r_I I_s\alpha_I}{II_s\alpha_{II}}$$

Above, we identified the limits beyond which what we called ‘absolute overaccumulation’ in one or the other department would occur; these were $r_I = \frac{I_v + I_s - II_c}{I_s\alpha_I}$ and $r_{II} = \frac{I_v + I_s - II_c}{II_s\alpha_{II}}$. Now we have the limits to what we can call ‘relative’ overaccumulation, i.e. within the limits of absolute overaccumulation an imbalance between rates of accumulation in the two departments. Relative overaccumulation in department I will occur when

$$\frac{I_v + I_s - II_c}{I_s\alpha_I} > r_I > \frac{I_v + I_s - II_c - r_{II}II_s\alpha_{II}}{I_s\alpha_I}$$

and in department II when

$$\frac{I_v + I_s - II_c}{II_s\alpha_{II}} > r_{II} > \frac{I_v + I_s - II_c - r_I I_s\alpha_I}{II_s\alpha_{II}}$$

(Evidently, and obviously, the limits to absolute overaccumulation are given when $r_{II}$ and $r_I$ respectively = 0.)

This is exactly what happened in the first reproduction scheme ($E^1$) set out by Marx here, and which he abandoned. The starting conditions were:

<table>
<thead>
<tr>
<th>Department</th>
<th>Capital ($\alpha$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>4,000 + 1,000 + 1,000 = 6,000</td>
</tr>
<tr>
<td>II</td>
<td>1,500 + 376 + 376 = 2,252</td>
</tr>
</tbody>
</table>

A rate of accumulation of 0.5 was assumed in both departments. Hence $I_d^{[c]} = 400$, $I_d^{[v]} = 100$, $II_d^{[c]} = 150$ and $II_d^{[v]} = 38$. But accumulated reproduction cannot take place in both departments under these circumstances: either department I reproduces, or department II does. According to the relations between the rates of accumulation worked out above, if we set $r_I$ to 0.5, $r_{II} \approx 0.33$; and if we set $r_{II}$ to 0.5, then $r_I \approx 0.44$. Thus, either department I maintains a rate of accumulation of 0.5 come what may, successfully reproducing on an expanded scale, but, as a consequence of relative overaccumulation here department II is left with unsold surplus product (to the value of 63) with no prospect of it being realised in the next (or any other) production period; or department II maintains a rate of accumulation of 0.5 come what may, in which case overaccumulation here leads to department I being left with money capital (62, obtained in the sale of means or production to department II to be accumulated as constant capital) it is unable to capitalise, i.e. convert into extra variable capital, because of a lack of constant capital to accompany it. These two cases of relative overaccumulation respectively are illustrated in figures 11 and 12.

Hence, if what in practice determines the rate of accumulation in general are conditions of competition between capitals, but what conditions what rate of accumulation is possible in a given sector is not only the conditions of production in that sector but also the aggregate rate of accumulation in the other sector, then it is reasonable to
assume that conditions of disproportion caused by relative accumulation in one or the other sector between sectors will prevail. The interesting question in this case is what the mechanisms to correct such disproportion would be.

**Figure 11**: scheme $E^1$: relative overaccumulation in department I

**Figure 12**: scheme $E^1$: relative overaccumulation in department II
4 What is not in the reproduction schemes

(a) Increase in labour productivity

I argued at the beginning of these notes that, on the basis of circumstantial evidence, it would be reasonable to suggest that Marx’s principle intention in setting out the reproduction schemes was to critique Adam Smith’s ‘dogma’ (that the value of commodities in its totality could be ‘resolved’ into ‘revenue’, that is profit and wages) and not to model the workings of an actual capitalist economy. Subsidiary evidence for this proposition is given by what is not included in the reproduction schemes.

In her introduction to Rosa Luxemburg’s *The Accumulation of Capital*, in which Luxemburg highlights a problem of demand for the product of department II (which I shall return to), Joan Robinson asked: ‘[i]f accumulation […] take[s] place, demand will absorb output, as the model [of Marx’s] shows, but what is it that makes accumulation take place?’

In volumes 1 and 3, Marx makes it clear that the capitalist (all capitalists), and once the limits of increasing the rate of absolute surplus-value have been reached, capitalists are hence driven to increase the productivity of labour through the adoption of more productive techniques of production, principally through the adoption of more productive machinery (fixed constant capital). Given the adoption of more productive production techniques, a given quantity of labour-power will transform a greater quantity of raw material (circulating constant capital) into finished commodity product. This is what drives capitalists to accumulate a part of their surplus-value: new and more productive machinery will tend to be more expensive, and, in any case, a greater mass (and value) of raw material is bought and worked up by a given mass of labour-power. Evidently, as this happens, the value composition of capital will change: the ratio of the value of constant capital laid out will rise as against the value of labour-power deployed.

But as new productive techniques become generalised, per-unit commodity prices in general fall, cheapening both the elements of constant capital (fixed and circulating) and the means of subsistence; in the case of the latter, if real wages remain constant, the value of labour power will fall. Nevertheless, although the cheapening of the elements of constant capital will counteract the direct effect of a rise in labour productivity on the value composition, because this cheapening is subsequent in time in terms of periods of production to the more productive techniques that cause the cheapening in the first place, the counteracting effect, assuming the rise in labour productivity is continuous, can only lessen, and not cancel out (let alone reverse) the rise in the value composition technological progress provokes. It is on this basis that Marx argues in *Capital* volume 3 the long-run tendency of a decline in the rate of profit.

How might the reproduction schemes of volume 2 appear if we factor in a rising productivity of labour and its effects?

Take the following example.

\[
\begin{align*}
\text{I} & : 1,500_c + 1,000_v + 1,000_s = 3,500 \\
\text{II} & : 1,500_c + 1,000_v + 1,000_s = 3,500 \\
\end{align*}
\]

Both departments are the same size and have the same value composition. If we assume a rate of surplus-value of 50% in both, and set the rate of accumulation of surplus-value in department I to \( \frac{250}{600} \approx 0.42 \), so that (given that \( r_I = \frac{I_v + I_s - \Pi_I \alpha_{II}}{I_c \alpha_I} \) and \( r_{II} = \frac{I_v + I_s - \Pi_{II} \alpha_{I}}{I_c \alpha_{II}} \), and that \( I_v + I_s = 500 \)) \( r_I = r_{II} \), then both departments will grow at exactly the same rate, like this:


Now let us factor in a continuous increase in the productivity of labour and the effects of this, in this way:

- If we imagine a 10% increase in the productivity of labour from period $P_{t+1}$ and on, in each production period the value of the constant capital set in motion by labour-power increases by a factor of 1.1.

- But, given that this increase in labour productivity will cheapen the elements of constant capital, but also given that the rise in labour productivity is continuous, from period $P_{t+2}$ and on we shall only increase the value of the constant capital set in motion by labour-power by a factor of 1.05.

- Given that the rise in productivity of labour will also result in a fall in the means of subsistence, we shall increase the rate of surplus-value by a factor of 1.05 for each production period from $P_{t+2}$ inclusive, while maintaining the value of $s + v$ the same as it would have been without this change (i.e. maintaining new value added in production, in other words the mass of labour-power deployed, the same as it would otherwise have been).

- Finally, we shall maintain the rate of accumulation in department I at $\frac{280}{600} \approx 0.42$.

The results are given in the table on the next page. What is immediately striking is the way that department I now grows more rapidly than department II: as one might intuitively expect, the greater demand for means of production provoked by a rising labour productivity is reflected in a differential growth pattern between the two departments. Although, by maintaining the rate of accumulation in department I as the independent variable, and therefore maintaining it higher than that in department II, perhaps this is not as significant as it might first appear. (When we take the rate of accumulation in department II as determining, the system almost immediately (by period $P_{t+4}$) moves into crisis: given that what is accumulated in department I as constant capital is the surplus commodity product in form of means of production left in department I after the subtraction of the total demand of department II for means of production is, i.e. $I_I - (II + r_{II}x_{II} - I_I)$, if department II grows too rapidly, $I_I < (II + r_{II}x_{II} - I_I)$ and not only is department I unable to supply department II with sufficient means of production but it is also unable to accumulate itself. The only way to guarantee the reproduction of the system in this case is to reduce the rate of accumulation in department II steadily, such that we have a pattern of accumulation in all respects effectively similar to that obtaining when department I determined accumulation.)
### Table 4: Two equal departments with technical progress

If we take scheme $E^3$ and make the same assumptions of technical progress and its consequences, we see another interesting effect (again, for the reasons just set out, we maintain determination by department I, maintaining here a rate of accumulation of 0.5).

![Table 4](image)

### Table 5: $E^3$ with technical progress

![Table 5](image)
Again, department I grows faster than department II (and enjoys a higher rate of accumulation). But once we reach period $P_{t+23}$, $I_v + I_u + I_a[v] > II_c$ (this is indicated by the negative numbers in the rate of accumulation and growth columns); what this means is that department II cannot now realise all of its commodity product $II_c$, and cannot reproduce its constant capital (a pattern of accumulation illustrated in figure 9 above). (This lack of demand for the commodity product of department II as a result of overaccumulation in department I as a consequence of a rising productivity of labour is exactly the problem that Rosa Luxemburg – albeit in a different way – identified in her *Accumulation of Capital*.\(^{62}\) For reproduction to occur under these conditions, the rate of accumulation in department I must be held above that in department II (either by allowing the former to determine the latter, or, in the contrary case, by constantly reducing the latter) – otherwise we cannot maintain $I_v \geq (II_c + r_{II}II_u - I_v)$ – and, seemingly inevitably, at a given point (in the scheme illustrated in table 4 above in period $P_{t+53}$) $I_v + I_u + I_a[v] > II_c$ and there is insufficient demand to realise all of the product of department II. Under the conditions thus far established there appears no way out of this problem.

(b) Mobility of capital

Thus far we have (implicitly) assumed that realised surplus-product (surplus-value in money form) can only be reinvested or consumed in the department in which it was produced. But, as Joan Robinson points out in her introduction to Luxemburg, there is no real reason for doing this. Savings out of the surplus accruing in each department (producers’ and consumers’ goods) are always invested in capital in the same department. There is no reason to imagine that one capitalist is linked to others in his own department more than to those in the other department, so the conception seems to be that each capitalist invests his savings in his own business. There is no lending by one capitalist to another and no capitalist ever shifts his sphere of operations from one department to another. This is a severe assumption to make even about the era before limited liability was introduced, and becomes absurd afterwards.\(^{63}\)

We have also been putting quite severe limits on the rate of accumulation (and in certain of the cases above the capitalists’ consumption fund has skyrocketed as a consequence). How much of their realised surplus-product do real capitalists accumulate in the real world? A fair answer would be, within certain limits, as much as they can.


\(^{63}\) *The Accumulation of Capital*, p. xxv. And as Robinson quite correctly points out, without mobility of capital between sectors there is no mechanism for the equalisation of the rate of profit.

Incidentally, Luxemburg is mistaken on the following point: ‘It is quite legitimate to suppose that under the technical conditions of progressive accumulation, society would invest ever increasing portions of the surplus value earmarked for accumulation in Department I rather than in Department II. [...] Yet this assumption is possible only so long as we envisage the surplus value earmarked for capitalisation purely in terms of value. The diagram, however, implies that this part of the surplus value appears in a definite material form which prescribes its capitalisation. Thus the surplus value of Department II exists as means of subsistence, and since it is as such to be only realised by Department I, this intended transfer of part of the capitalised surplus value from Department II to Department I is ruled out, first because the material form of this surplus value is obviously useless to Department I, and secondly because of the relations of exchange between the two departments which would in turn necessitate an equivalent transfer of the products of Department I into Department II. It is therefore downright impossible to achieve a faster expansion of Department I as against Department II within the limits of Marx’s diagram.’ (*The Accumulation of Capital*, pp.320-21)

What is being confused here is the *realisation* of the surplus-product – its conversion into money – and its *accumulation* – the conversion of this money into additional factors of production (constant capital and labour-power). The realisation of the surplus-product of each department is indeed dependent on (amongst other things) its material form, for means of production are means of production and means of subsistence are means of subsistence. But once this realisation takes place – if it does, and the limits for the possibility of realisation have been set out above – then by definition the material form of the surplus *prior* to its realisation no longer material. ‘Since money does not reveal what has been transformed into it, everything, commodity or not, is convertible into money. Everything becomes saleable and purchasable.’ (C1, p. 229)
Let us then relax these two assumptions, and see what happens, using as a model the equal department scheme set out above. The procedure we shall follow is this:

- The total surplus-value is summed and made available for accumulation in both departments.

- The capitalists’ consumption fund in both departments is allowed to rise by 10% per period; the total surplus available for accumulation is thus set at the total surplus less the combined capitalists’ consumption fund.

- Given the rise in the value composition – set as above – the total surplus available for accumulation is reduced by that amount necessary for the greater value of constant capital that the fixed amount of labour-power which we assume demand (but it has of course already been augmented by the saving in variable capital caused by the rise in the rate of surplus-value).

- The resulting surplus-value available for accumulation (the total surplus less the capitalists’ consumption fund less the constant capital that has necessarily to be accumulated as a consequence of the rising value composition) is distributed as follows:
  - The shortfall between $II_c$ and $(I_v + I_u)$ is accumulated in department II as constant capital (and the necessary variable capital is accumulated too).
  - What remains is distributed according to the fact that a value equivalent to that accumulated in department I as variable capital has to be accumulated in department II as constant capital, and that this last requires a given quantity of labour-power.

The pattern of accumulation that results is shown in the table on the following page; there are a number of features displayed worth taking note of.

- First, and most strikingly, there are now no more ‘disproportion crises’: both departments grow, and all product is realised; a state of affairs, it appears, that could continue in perpetuity.64

- Department I again enjoys a greater rate of growth, and this is now independent of our fixing an arbitrary rate of accumulation in either one department or the other.65

- But if there are now no longer necessary ‘disproportion crises’, this does not mean that all is entirely well, for there is another cloud looming on the horizon. We see already in the lower part of the table that rate of profit is falling in both departments, that the rate of growth is falling in department II and slowing in department I. By period $P_{t+40}$ the rate of profit in department I stands at 22.6% and in department II 21.2%, while the rate of growth stands at 21.9% and 16.9% in departments I and II respectively.

These, however, are problems that we will have to wait until volume 3 to investigate.

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64 Although by period $P_{t+76}$ the demand for extra constant capital to be accumulated as a consequence of the rise in the value composition exceeds the total surplus-value from which it would have to be accumulated.

65 ‘[I]t is clear that a quicker growth of constant as compared with variable capital, i.e. the progressive metamorphosis of the organic composition of capital, must take the material form of faster expansion of production in Department I as against production in Department II.’ (The Accumulation of Capital, p.320)
Table 6: Two equal departments with technical progress and mobility of capital

<table>
<thead>
<tr>
<th>Production period</th>
<th>Department I</th>
<th></th>
<th></th>
<th>Department II</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>c</td>
<td>v</td>
<td>s</td>
<td></td>
<td>c</td>
<td>v</td>
</tr>
<tr>
<td>t</td>
<td>1500.0</td>
<td>1000.0</td>
<td>1000.0</td>
<td>0.60</td>
<td>416.7</td>
<td>583.3</td>
</tr>
<tr>
<td>t+1</td>
<td>1658.0</td>
<td>1138.7</td>
<td>1138.7</td>
<td>0.62</td>
<td>357.5</td>
<td>641.7</td>
</tr>
<tr>
<td>t+2</td>
<td>2173.5</td>
<td>1273.6</td>
<td>1334.2</td>
<td>0.63</td>
<td>534.3</td>
<td>705.8</td>
</tr>
<tr>
<td>t+3</td>
<td>2619.6</td>
<td>1470.5</td>
<td>1607.2</td>
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(c) Realisation of the commodity product

On a number of occasions, David Harvey has advanced the argument that there exists in Marx’s reproduction schemes a problem of ‘effective demand’:66 that if, at the beginning of a given production period, capitalists advance $c$ and $v$ as capital, and at its end they are left with a commodity product to the value of $c + v + s$, then there is no demand in the system to realise the surplus product $s$. In order to realise the surplus product the capitalists have to borrow, but this borrowing is realised on the strength of the realisation of the following period’s surplus product. But a similar demand problem exists in this period as well: if only $c'$ and $v'$ are cast into circulation at the start of this, next, period, then there is again no demand to realise this period’s surplus product either. Hence, robbing Peter to pay Paul, capitalists have continually to realise the surplus product of one period against the surplus product of the next.

This observation also has an explicative function. If capitalists can only realise the surplus product of one period with money borrowed against the realisation of that of the next, they are forced by this circumstance not only to continue production but to continue production an expanded and expanding scale, to produce more surplus-value in one period than they produced in the previous. This is fundamentally why capitalists accumulate realised surplus product as new capital.⁶⁷

The problem with Harvey’s argument is that it conceives of demand only at the level of money: if $x$ quantity of money is cast into circulation at the beginning of a production period then only $x$ value of commodity product can be realised at its end. But where does the money that is cast into circulation (to make a purchase) come from? It comes from a previous sale. For reproduction to occur, sales and purchases have to match up; money cast in has, ultimately, to return.⁶⁸ Marx’s criticism of ‘Say’s law’ in volume 1 is not that it says that sales and purchases have to match up, it is that the law states that they necessarily will.⁶⁹ For commodity product – including the surplus product – to be realised, all that is necessary is that it has to be bought; for it to be bought, what is necessary is that product elsewhere be sold. This is the content of Marx’s comment in the Grundrisse that ‘[t]he surplus-value created at one point [in production] requires the creation of surplus value at another point […]’.⁷⁰ Although the form of demand is monetary, the demand for a given value of commodity product does not originate with the money with which it is purchased but with the existence elsewhere in social production of an equivalent value against which it is exchanged.

If capitalist $A$ holds €1,000 of commodity product, a capitalist $B$ (or $B$ and $C$ and $D$, etc.) also holding €1,000 of commodity product is a necessary precondition for $A$ to realise her product. $A$ might borrow €1,000 (from $C$, in reality the credit system), use it to buy $B$’s product, and then $B$ can use the same €1,000 to buy $A$’s product, whereupon $A$ can return the money to $C$. The product of both $A$ and $B$ has been successfully realised, and may now be (productively or unproductively) consumed. But if $A$ held €1,000 of product, and $B$ only €500, half of $A$’s product would not be realised, at least not immediately, independently of the available existence of money, because of the lack of value equivalence demand.

Money is, thus, necessary to the process; but it is, in a sense, how demand manifests itself, and Marx’s assumption has been that money is already in existence in the system in the form of hoards (the credit system). In addition, there is no reason for the quantity of money necessary to realise a given value of product be equal to the commodity value to be realised: Marx has been at pains to point out that, precisely because as commodities drop out of circulation and into (productive and unproductive) consumption money remains to realise further exchanges, a relatively small amount of money can realise an enormous amount of circulating commodity value. In the example above, €1,000 was sufficient to realise €2,000 of commodity product, but €100, passing twenty times between the capitalists, would also realise the same end.

So, although demand takes the form of monetary effective demand, this latter is premised on the existence of ‘value equivalence’ demand. Understood in this way there is no difficulty in understanding the realisation of the surplus product. If the total social product is equal to $c + v + s$ then the total value equivalence demand is equal to $c + v + s$ as well by definition: the workers buy back $v$ with their wages while the capitalists sell and buy $c$ and $s$ amongst themselves. The complication that we meet in the reproduction schemes is the (use-value) division of the social product into means of production and means of subsistence. That part of the total social product that is to be

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⁶⁷ This argument also has an explanatory function in relation to crises (the present one included) as well. If capitalists, belaboured by an increasing weight of debt, are forced, to service this debt, to increase production in value terms year on year, then, should production not grow in sufficient scale, or should – which is just one form of manifestation of this – capital be attracted away from material production to ‘fictitious’ fields of investment (asset markets and the like) the volume of debt becomes unserviceable.

⁶⁸ ‘In order that the exchange should take place normally, it has to be assumed that the one-sided purchase by II, is equal in value to its one-sided sale, and similarly that the one-sided sale of I, to II, […] is equal to its one-sided purchase from II, […] Otherwise, […] reproduction would be disrupted; the one-sided purchase at one point must be covered by a one-sided sale at another.’ C2, p. 570.

⁶⁹ C1, pp. 208-9.

unproductively consumed has to exist in its entirety in the form of means of (capitalists’) subsistence and that part to be productively consumed in the form of means of production and money (to pay wages). Hence the necessity of exchanges between the departments. And what the foregoing has been about is the conditions of these exchanges necessary for the realisation of the commodity product, how a part of the surplus product of each department is exchanged against the product of the other, and the value equivalence conditions necessary for these exchanges to take place (and also those value equivalence conditions in which a part of the product cannot be realised). There is, in this sense, no necessary problem of ‘effective demand’.

In the reproduction schemes we have been considering the two departments as ‘collective capitalists’: in the exchange I_u ↔ II_c, for example, we have imagined one of our collective capitalists advancing money (derived from the credit system) to buy the product of the other and then the other using the money to buy the product of the first. But if we conceive of the departments as consisting in capitalists, then, while I’ may advance money to buy the product of II’, II’ may well use the money advanced to buy the product of I’, who may then buy the product of II”, who then buys from I””, and so on. Our assumption is that the aggregate sales and purchases if I’, I”, II’, II””, etc. will balance out in the end, but, as we saw in volume 1, not only is there no guarantee that this will be the case, it is also evidently impossible for these exchanges to be prefigured before the case such that this actually be the case. This is a consideration, the permanent actuality of the possibility of crisis following from the nature of commodity circulation itself, however, that lies outside of level of abstraction of the reproduction schemes.

There are further aspects to the problem of realisation that are also not addressed. For a given portion of the commodity product to be realised it is not merely sufficient that there be a monetary demand backed by value equivalence demand for it; there also needs to be a use-value demand – the product to be realised must match a given need or desire for it, and, if it does not, irrespective of the existence of monetary and value equivalence demand, it will not be realised.

But even this is insufficient: for even if there exists a value equivalence demand, backed by money, and a use-value demand for the product, this last needs to exist on the market in the right quantity and at the right price in function of the social demand for it. This becomes especially important in conditions of rising productivity of labour: when a greater mass of product is being produced, even if at a lower unit cost, there is no guarantee before the fact that this greater mass will find the social demand necessary for its realisation.

Here are three aspects of the problem of realisation of the commodity product – the non Say’s Law like nature of commodity circulation, the necessity of the existence of use-value demand, and the necessity of the existence of social demand – all of which suggest the possibility of the non-realisation of the commodity product and hence the permanent actuality of the possibility of crisis that are not addressed, given the level of abstraction at which Marx is working, in the reproduction schemes.

There is also a further aspect. Earlier we identified a specific problem in relation to the realisation to that part of the social product of one period equivalent to the surplus-value to be accumulated as variable capital in the next period. Of the commodity product of department II that is consumed outside this department (i.e. by the capitalists and workers of department I), that which is equivalent to the variable capital laid out by department I, and that equivalent to the surplus product to be consumed unproductively by the capitalists of department I, is realised at the end of the production period during which it is produced by the wages laid out at the beginning of the period by the capitalists of department I and by the money laid out at the beginning of the period by the capitalists of department II on constant capital (money which returns as the capitalists of department I buy means of subsistence to consume unproductively) respectively. But at the end of each production period the capitalists of department II lay out money (to buy means of production), money which originates as money which is advanced by department I as accumulated variable capital. But if this money is advanced against an equivalent part of department II’s product,
this portion of product will not be realised until this money, in the form of wages, is spent by the workers who receive it, but these workers will not use it to buy means of subsistence until the end of the next period.

In the same way, that part of the product of department II equivalent to the variable capital accumulated at the end of one period can only be realised at the end of the period subsequent to its production.

In other words, at the end of each production period department II holds unrealised commodity product, and an equivalent money debt, equivalent to the variable capital to be accumulated in both departments. The product is realised, and the debt repaid, but with a delay.

But, as we have seen, at the end of each production period, department II reproduces its constant capital in exchanges against $I_v$ and $I_u$ (the excess between this latter quantity and II, being accumulated in department II as constant capital); since, for any given period, $I_v$, the money laid out as variable capital, is the sum of the previous period’s $I_v$ and $I_{a(v)}$, if the previous period’s $I_{a(v)}$ is required to realise part of the previous period’s commodity product then the next period’s $I_v$, necessary to realise the next period’s product, is short by this amount.

By the same token, at the beginning of each production period the capitalists of department II lay out money as variable capital, money which returns as the workers buy product $II_v$ with their wages. But a part of the product $II_v$ needs to be accumulated as variable capital; this part of the product also cannot be realised until the ‘extra’ (accumulated) are spent at the end of the next period. But in this (subsequent) period the variable capital laid out at its beginning is required to realise the whole of the product $II_v$, but this variable capital is the sum of the variable capital laid out in the previous period and that surplus-value accumulated as labour-power. Hence the sum available to realise $II_v$ in one period is again short by the $II_{a(v)}$ accumulated at its beginning, for this last is required to realise that part of department II’s product unrealised in the previous period.

Taking scheme $E^2$ as an example, at the end of $Pt$, department II holds commodity product to the value of $II_{a(v)} (= 100)$ and $II_{a(v)} (= 50)$, and a debt (money borrowed to buy means of production to the value of $I_{a(v)}$ and to lay out variable capital $II_{a(v)}$ ) of 150.

At the end of $Pt+1$, $II_{a(v)}$ is reproduced against $I_{a(v)}$ and $I_{a(v)}$, but since $I_{a(v)} = I_{a(v)} + I_{a(v)}$, and $I_{a(v)}$ is required to complete the realisation of $II_v$, a part of $II_{a(v)}$ goes unsold.

In addition, the wages $II_{a(v)}$ are required to realise commodity product $II_{a(v)}$, but of these wages, which = $II_{a(v)} + II_{a(v)}$, $II_{a(v)}$ is required to realise the as yet unsold part of $II_v$.

So, although at the end of $Pt+1$ all of department II’s product produced in $Pt$ is realised (and accompanying dept paid off), an equivalent value of product ($II_{a(v)}$ ) remains unsold; money needs to be advanced (borrowed) to complete the realisation of the product of department I.

In addition to this, though, the same problem that occurred in $Pt$ occurs in $Pt+1$ : department II is left with commodity product (and therefore an equivalent debt to the value of $I + II_{a(v)}$, product that will be realised in $Pt+2$, but against which money must be advanced (as $II_{a(v)}$ and $II_{a(v)}$ ). Hence the problem of the delay of the realisation of the commodity product of department II is not only continuous, but cumulative. At any given period

of production $Pt+n$, the value of the unsold product (and accompanying debt) stands at $\sum_{j=0}^{n} [I + II_{a(v)}]$.

What determines the value of the unsold product – which is the same as what determines the value of the accumulated variable capital – is the rate of accumulation and the value composition of total social production; in the case of $E^2$ (as in that of $E^3$), these values are fixed and stable; as a consequence, in the case of $E^2$, there is an
infinite egress towards a per period rate of growth of unrealised product of 10% and the total value of unrealised product forming 66.3% of the total product of department II.

Once we factor in increasing labour productivity and its consequences, then the weight of $v$ falls progressively, such that the size of the unrealised product as a proportion of the total product of department II eventually falls too (in the case of the scheme with two initially equal departments with technical progress and mobility of capital illustrated in table 6 above, the proportion peaks in period $P_{t+16}$ and thereafter falls).

The resulting glut of means of subsistence is not a consequence of ‘overproduction’ (or ‘underconsumption’), but rather of a delay in the realisation of commodity. Nevertheless, there is a glut.